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14. ABSTRACT In this project: -- We investigated the fundamental question of how electromagnetic waves propagate in an important class of inhomogeneous metamaterials, with material properties gradually changing from positive to negative values ("transition metamaterials"). We discovered strongly polarization sensitive anomalous field enhancement near the zero refractive index point under oblique incidence of the wave on a realistic, lossy transition metamaterial layer					
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Report Title

NONLINEAR OPTICS IN NEGATIVE INDEX METAMATERIALS: FINAL REPORT

ABSTRACT

In this project:

- We investigated the fundamental question of how electromagnetic waves propagate in an important class of inhomogeneous metamaterials, with material properties gradually changing from positive to negative values ("transition metamaterials"). We discovered strongly polarization sensitive anomalous field enhancement near the zero refractive index point under oblique incidence of the wave on a realistic, lossy transition metamaterial layer potentially enabling a variety of applications in microwave, terahertz, and optical metamaterials, including subwavelength transmission and low-intensity nonlinear optical devices.
 - We developed a generalized analytical model and solutions for nonlinear wave propagation in waveguide couplers with opposite signs of the linear refractive index, non-zero phase mismatch between the channels, and arbitrary nonlinear coefficients. These results offer a practical tool for designing novel metamaterials based couplers based on either double-negative or strongly anisotropic metamaterials that are likely to enable ultra-compact optical storage and memory components for photonics on a chip applications.
 - We proposed and investigated in detail a structure consisting of a nonlinear core with focusing or de-focusing Kerr nonlinearity and graded-index shell designed using transformation optics that can be switched from a concentrator squeezing light into the core to a variable focus lens by varying the intensity of incident light.
 - As an extension of this project, we performed preliminary studies of the wave and ray properties of electromagnetic wave interactions in two-dimensional microcavities that contain a combination of PIM and NIM with negative dielectric permittivity and magnetic permeability. These studies indicate that NIM-PIM disk resonators can be specifically designed to emit intense radiation in a controlled manner. Our preliminary results show that the new NIM-PIM micro-cavities show evidence for the chaotic behavior and scar modes. Future studies include the investigations of localized gain for selective amplification, PIM-NIM cavities with loss and/or gain and optical nonlinearities.
 - We designed and experimentally demonstrated optical fiber-coupled magnetic metamaterials integrated on the transverse cross-section of an optical fiber. Such fiber-metamaterials integration may provide fundamentally new solutions for photonic-on-a-chip systems for sensing, subwavelength imaging, image processing, and biomedical applications.
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Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

<u>Received</u>	<u>Paper</u>
2011/09/05 1: 6	Yinnan Chen, DongHo Lee, Natalia Litchinitser, Alexander Cartwright, Jinwei Zeng, Xi Wang, Gayatri Venugopal. Optical Fiber Metamagnetics, Optics Express, (10 2011): 0. doi:
2011/09/05 1: 5	Ethan Gibson, Ildar Gabitov, Andrei Maimistov, Natalia Litchinitser. Transition Metamaterials with Spatially Separated Zeroes, Optics Letters, (08 2011): 0. doi:
2011/09/05 1: 4	Gayatri Venugopal, Zhaxylyk Kudyshev, Natalia Litchinitser. Asymmetric Positive-Negative Index Nonlinear Waveguide Couplers, IEEE Journal of Selected Topics in Quantum Electronics, (10 2011): 0. doi:
2011/09/05 1: 1	A.D. Boardman, V.V. Grimalsky, Y.S. Kivshar, S.V. Koshevaya, M. Lapine, N.M. Litchinitser, V.N. Malnev, M. Noginov, Y.G. Rapoport, V.M. Shalaev. Active and tunable metamaterials, Laser & Photonics Reviews, (03 2011): 0. doi: 10.1002/lpor.201000012
2011/09/05 1: 2	Ethan A Gibson, Matthew Pennybacker, Andrei I Maimistov, Ildar R Gabitov, Natalia M Litchinitser. Resonant absorption in transition metamaterials: parametric study, Journal of Optics, (02 2011): 0. doi: 10.1088/2040-8978/13/2/024013
2011/09/05 1: 3	I Mozjerin, EA Gibson, EP Furlani, IR Gabitov, NM Litchinitser. Electromagnetic enhancement in lossy optical transition metamaterials, Optics Letters, (10 2010): 3240. doi:

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(b) Papers published in non-peer-reviewed journals (N/A for none)

Received

Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

1. N. M. Litchinitser, Photonic Metamaterials: From Linear to Nonlinear Optics, Invited seminar at Indiana University, Physics Dept., Oct. 21, 2011.
2. N. M. Litchinitser, Photonic Metamaterials: From Linear to Nonlinear Optics, Invited seminar at SMU, Dept. of Mathematics, Nov. 17, 2011.
3. N. M. Litchinitser, Photonic Metamaterials: From Linear to Nonlinear Optics, Invited seminar at UT Arlington, EE Dept., Nov. 18, 2011.
4. N. M. Litchinitser Transition Metamaterials, SIAM Conference on Nonlinear Waves and Coherent Structures,” (NW10), Philadelphia, PA 2010.
5. N. M. Litchinitser, Photonic Metamaterials: From Linear to Nonlinear Optics, Technical University of Denmark, 2010.
6. N. M. Litchinitser, Photonic Metamaterials: From Linear to Nonlinear Optics, Norfolk State University, 2010.
7. N. M. Litchinitser, Photonic Metamaterials: From Linear to Nonlinear Optics, University of Rochester, Institute of Optics, 2010.
8. N. M. Litchinitser, Waves in Graded Index and Nonlinear Metamaterials, 6th Annual CRI Summer Conference on Metamaterials, Charlotte, NC, 2009.
9. N. M. Litchinitser, Graded-index plasmonic metamaterials: linear and nonlinear wave propagation,” SPIE Optics & Photonics Conference, Plasmonics Workshop, San Diego, CA, 2009.
10. N. M. Litchinitser, Nonlinear effects in positive-negative-index guided wave structures, SPIE Optics & Photonics Conference, Photonic Metamaterials Workshop, San Diego, CA, 2009.
11. N. M. Litchinitser, E. A. Gibson, G. Venugopal, Z. Kudyshev, A. Pandey, Guiding waves with nonlinear metamaterials, SPIE Optics & Photonics, Paper 8093?11, San Diego, CA, August 21?25, 2011.
12. A. K. Popov, I. R. Gabitov, N. M. Litchinitser, A. V. Kildishev, V. M. Shalaev, Nonlinear and active metamaterials, SPIE Optics & Photonics, Paper 8093-05, San Diego, CA, August 21-25, 2011.
13. Metamaterials: A gateway to new science and applications of light, The Fifth International Conference on Nanophotonics, Shanghai, May 22-26, 2011.
14. N. M. Litchinitser, I. Mozjerin, T. Gibson, M. Pennybacker, I. R. Gabitov, Transition Metamaterials, SIAM Conference on Nonlinear Waves and Coherent Structures (NW10), Philadelphia, PA, USA (2010).
15. N. M. Litchinitser, I. Mozjerin, T. Gibson, E. P. Furlani, I. R. Gabitov, Electromagnetic Enhancement in Lossy Optical Transition Metamaterials, Laser Physics Conference, Brazil (2010).
16. T. Gibson, I. Mozjerin, and N. M. Litchinitser, Transition Metamaterials: Theory and Design Optimization, Laser Physics Conference, Brazil (2010).
17. N. M. Litchinitser, T. J. Gibson, I. R. Gabitov, and V. M. Shalaev, Graded?index metamaterials: From linear to nonlinear optics, 3rd International Congress on Advanced Electromagnetic Materials in Microwaves and Optics, London, UK (2009).
18. N. M. Litchinitser, A. I. Maimistov, I. R. Gabitov, R. Z. Sagdeev, and V. M. Shalaev, Waves in Graded Index and Nonlinear Metamaterials, 6th Annual CRI Summer Conference on Metamaterials, Charlotte, NC (2009).
19. N. M. Litchinitser, T. J. Gibson, I. R. Gabitov, and V. M. Shalaev, Graded-index plasmonic metamaterials: linear and nonlinear wave propagation, SPIE Optics & Photonics Conference, Plasmonics Workshop, San Diego, CA (2009).
20. N. M. Litchinitser, Y. Xiang and V. M. Shalaev, Nonlinear effects in positive-negative-index guided wave structures, SPIE Optics & Photonics Conference, Photonic Metamaterials Workshop, San Diego, CA (2009).
21. A. Pandey, E. A. Gibson, N. M. Litchinitser, Transition metamaterials in materials with hyperbolic dispersion, SPIE Optics & Photonics, Paper 8093?22, San Diego, CA, August 21-25, 2011.
22. N. M. Litchinitser, Nonlinear and Guided Waves Optics in Metamaterials, Workshop on Metamaterials and Plasmonics: Novel Materials, Designs, and Applications, Buffalo, NY, May 16-17, 2011
23. N. M. Litchinitser, Metamaterials as a Potential New Platform for Nonlinear and Spin-Optics, International Workshop "Beyond the Imagination of Nature: Spin, Quantum Optics and Metamaterials," Buffalo, NY, September 19?20, 2011
24. N. M. Litchinitser, T. Gibson, G. Venugopal, M. Pennybacker, I. Mozjerin, I. R. Gabitov, Nonlinear Optics in Transition and Negative Index Metamaterials, Special Session on Strongly-nonlinear Phenomena: Theory and Applications to Nonlinear Optics, Hydrodynamics, Bose-Einstein Condensation and Biology, Albuquerque, NM (2010).

Number of Presentations: 24.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):	
<u>Received</u>	<u>Paper</u>
TOTAL:	
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Peer-Reviewed Conference Proceeding publications (other than abstracts):	
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(d) Manuscripts	
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Books	
<u>Received</u>	<u>Paper</u>
2011/09/05 11:07	Natalia Litchinitser, Ildar Gabitov, Andrei Maimistov, Vladimir Shalaev. Tutorials in Metamaterials: Chapter : Linear and Nonlinear Metamaterials and Transformation Optics, CRC Press: Taylor & Francis Group, (10 2011)
TOTAL: 1	

Patents Submitted	
Bridging Fiber Optics with Metamaterials, Fiber Optic-Based Metamaterials	

Patents Awarded	
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Awards	
1. Elected a Fellow of the Optical Society of America, 2011	
“The OSA Fellow designation is awarded to select OSA Members who have made significant contributions to the advancement of optics. Candidates for Fellow Membership are nominated by peers who are themselves OSA Fellows. The OSA Fellowship Members Committee reviews each nomination and recommends candidates to the Board of Directors annually. The number of Fellows is limited by the Society's bylaws to be no more than 10% of the total OSA membership” (from http://www.osa.org/Membership/Member_Categories/Fellow/)	
2. Elected a Senior Member of IEEE (2011).	
3. Tenured professorship (2011).	

Graduate Students

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	Discipline
Ethan Gibson	0.50	
Apra Pandey	1.00	
Gayatri Venugopal	1.00	
Mehdi Pakmehr	0.10	
FTE Equivalent:	2.60	
Total Number:	4	

Names of Post Doctorates

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
Irene Mozjerin	1.00
FTE Equivalent:	1.00
Total Number:	1

Names of Faculty Supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	National Academy Member
Natalia Litchinitser	0.08	
FTE Equivalent:	0.08	
Total Number:	1	

Names of Under Graduate students supported

<u>NAME</u>	<u>PERCENT SUPPORTED</u>	Discipline
Yinnan Chen	0.25	Electrical Engineering
FTE Equivalent:	0.25	
Total Number:	1	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period:	1.00
The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:.....	1.00
The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:.....	1.00
Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):.....	1.00
Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:.....	0.00
The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense	0.00
The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:	1.00

Names of Personnel receiving masters degrees

<u>NAME</u>
Mehdi Pakmehr
Total Number:

1

Names of personnel receiving PhDs

NAME

Gayatri Venugopal

Ethan Gibson

Total Number:

2

Names of other research staff

NAME

PERCENT SUPPORTED

FTE Equivalent:

Total Number:

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Technology Transfer

FINAL REPORT

Natalia M. Litchinitser

I. Statement of the problem studied

Photonic metamaterials (MMs) are artificial nanostructures that emerge as a source of nearly unlimited opportunities for the realization of material properties that were not previously accessible, including positive, negative, and even zero indices of refraction. The emergence of metamaterials and, in particular, negative index metamaterials (NIMs) triggers reconsideration of many fundamental physical phenomena. Importantly, the majority of unique properties of NIMs stand out when NIMs are combined with conventional positive index materials (PIMs). Unique applications of MMs include super-lenses that beat the fundamental diffraction limit and enable high-resolution optical imaging, and cloaking devices that render macroscale objects invisible. In this project, we investigated fundamental linear and nonlinear phenomena in magnetic, negative-index and graded-index MMs. Designed using the technique of transformation optics; these structures enable a number of new regimes of light-matter interaction and potential applications in both linear and nonlinear optical regimes.

Highlights of the most important results:

- We investigated the fundamental question of how electromagnetic waves propagate in an important class of inhomogeneous metamaterials, with material properties gradually changing from positive to negative values ("transition metamaterials"). We discovered strongly polarization sensitive anomalous field enhancement near the zero refractive index point under oblique incidence of the wave on a realistic, lossy transition metamaterial layer potentially enabling a variety of applications in microwave, terahertz, and optical metamaterials, including subwavelength transmission and low-intensity nonlinear optical devices.
- We developed a generalized analytical model and solutions for nonlinear wave propagation in waveguide couplers with opposite signs of the linear refractive index, non-zero phase mismatch between the channels, and arbitrary nonlinear coefficients. These results offer a practical tool for designing novel metamaterials based couplers based on either double-negative or strongly anisotropic metamaterials that are likely to enable ultra-compact optical storage and memory components for photonics on a chip applications.
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- As an extension of this project, we performed preliminary studies of the wave and ray properties of electromagnetic wave interactions in two-dimensional microcavities that contain a combination of PIM and NIM with negative dielectric permittivity and magnetic permeability. These studies indicate that NIM-PIM disk resonators can be specifically designed to emit intense radiation in a controlled manner. Our preliminary results show that the new NIM-PIM micro-cavities show evidence for the chaotic behavior and scar modes. Future studies include the investigations of localized gain for selective amplification, PIM-NIM cavities with loss and/or gain and optical nonlinearities.
- We designed and experimentally demonstrated optical fiber-coupled magnetic metamaterials integrated on the transverse cross-section of an optical fiber. Such fiber-metamaterials integration may provide

fundamentally new solutions for photonic-on-a-chip systems for sensing, subwavelength imaging, image processing, and biomedical applications.

II. Summary of the most important results

Metamaterials constitute a new, 21st century area of research that is expanding fundamental knowledge of the behavior of electromagnetic wave propagation and present potential novel solutions to the realization of entirely new photonic functionalities such as sub-wavelength imaging, invisibility cloaking, and all-optical signal processing.

1.1 Field enhancement effects in near-zero refractive index MMs – towards low-intensity nonlinear optics

The emergence of negative index materials (NIMs) has given rise to numerous unusual phenomena that cannot be realized in conventional materials. These materials have revolutionized modern optics by providing unparalleled potential opportunities for designing novel applications, including nano-imaging and sensing devices, solar cells and light-emitting devices. Owing to the structural complexity of these composite materials, theoretical predictions and numerical analysis are the essential components of NIMs research.

Significant progress in the understanding of MM fundamentals and recent developments in fabrication technologies have given rise to the field of transformation optics. Transformation optics is based on MMs with a tailored spatial distribution of the refractive index, which can vary from positive to negative values.

While the optical properties and potential applications of uniform MMs with constant refractive indices have been studied in detail and are quite well understood, graded-index MMs – artificial materials with refractive indices gradually varying in space in a wide range from positive to zero to negative values – have received significantly less attention so far. The enormous potential of graded-index MM structures

was recently exemplified by the first experimental demonstration of an invisibility (cloaking) device. However, cloaking is just one of numerous prospective applications of these structures. Nevertheless, no fundamental physical models, design and optimization numerical tools, or experimental platforms exist to date to fully explore their unique properties. Our research is focused on the development of theoretical and numerical models for understanding, modeling and controlling linear and nonlinear interactions of light with graded-index photonic MMs and their device applications.

In particular, we predicted and investigated the resonant enhancement of electromagnetic waves propagating at oblique incidence in MMs near a point where the real part of the refractive index is zero, as shown in Fig. 1. This effect occurs for both TE and TM polarizations near the point where the refractive index changes its sign at it transitions through zero. Our model elucidates the unique features of the resonant enhancement in “positive-to-negative transition” MMs for a broad frequency range from microwaves to optics. We performed a

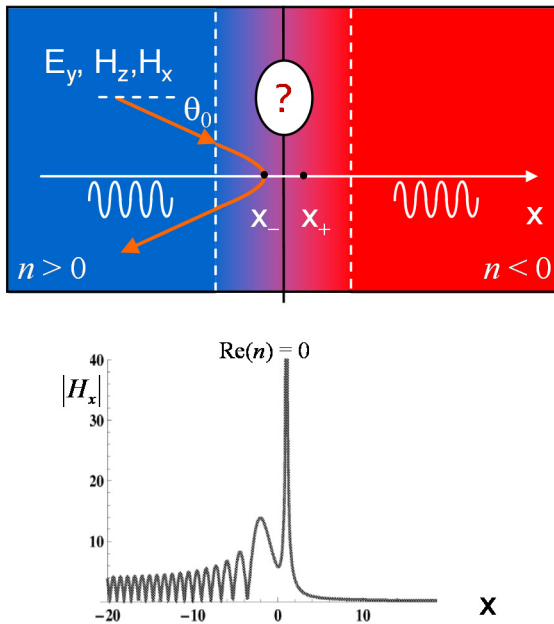


Figure 1. Upper plot: A schematic of transition metamaterial. Lower plot: Resonant field enhancement near zero-index transition.

detailed study of the effect of resonant absorption in transition MMs with various refractive index profiles. The results of this parametric study may be of considerable interest for a variety of MMs-based applications. Depending on the application, the amount of resonant absorption may be minimized or maximized by changing the parameters of the transition layer and more generally the spatial profiles of material parameters. For example, resonant absorption may affect the performance of a superlens with diffused boundaries or other structures based, for example, on doped semiconductors, and therefore, needs to be minimized. On the other hand, applications such as “perfect” absorbers or nanodetectors would benefit from the resonant absorption effect that in this case should be maximized. Moreover, our results could be of considerable importance in the context of transformation optics as the inherent near zero index transition phenomena in graded-index structures could significantly change spatial field distributions and lead to undesired absorption.

Also, we demonstrated that more complex refractive index distributions with, for example, two zero-index crossings (Fig. 2), may result in a formation of highly polarization sensitive resonant cavities. Potential applications of these local field enhancement effects include for low-intensity nonlinear optical devices, including switching and antenna applications. Our current research directions in this area include:

- i. Micro- to- nano-scale optical tapers: in this project, we design a micro- to- nano-scale optical taper: a device that guides and concentrates light to a sub-wavelength spot for applications including optoelectronic integrated circuits, sensors, and photovoltaics. This device would offer a viable solution to one of the major problems of modern optoelectronics – bridging the gap between micro-scale photonic waveguides (e.g., optical fibers) and nano-scale optical and electronic devices. In this project, we utilize the strong enhancement of the field near the zero-index point and an anisotropic refractive index profile designed via the transformation optics technique.
- ii. Novel nonlinear wavelength converters: this application relies on the predicted resonant field enhancement near the zero refractive index point in combination with the nonlinear response of the MMs host medium. Our research concentrates on applications based on both second- and third-order nonlinearities. An important advantage of this approach over the existing solutions is the potentially low

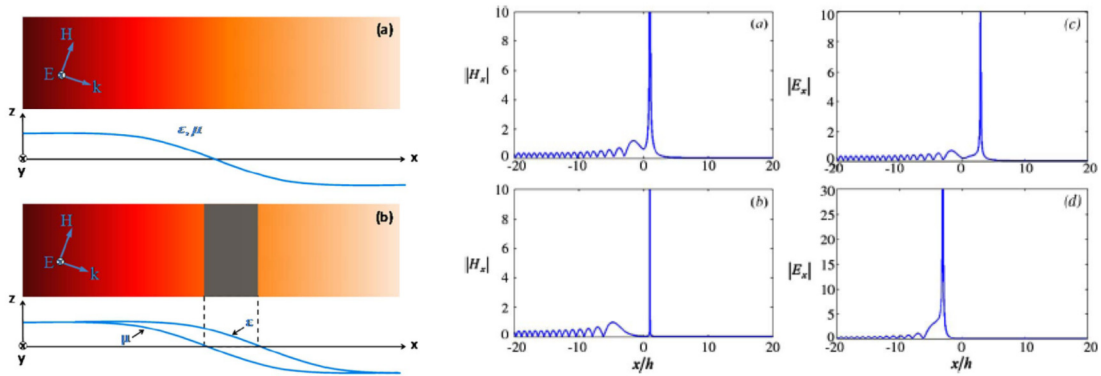


Figure 2. Left: (a) Schematics of a transition metamaterial with ϵ and μ crossing zero at the same point. (b) Schematics of a transition metamaterial with ϵ and μ crossing zero at two different points in space. Right: Figures (a) and (b) show the absolute values of the magnetic field component for the TE wave H_x as functions of x/h for two different values of the parameter l [$l = 2$ (a) and $l = -4$ (b)]. Figures (c) and (d) show corresponding results for the TM polarization. In this case, the longitudinal component of the electric field (E_x) is enhanced and, therefore, we show the absolute values of the electric field component E_x as functions of x/h for two different values of the parameter l [$l = 2$ (a) and $l = -4$ (b)].

input power requirements owing to the strong field localization effect that enables nonlinear effects at moderate intensities. New regimes of nonlinear conversion are expected owing to unusual phase-matching conditions that may be enabled at the PIM-NIM interface.

1.2 Anisotropic transition metamaterials

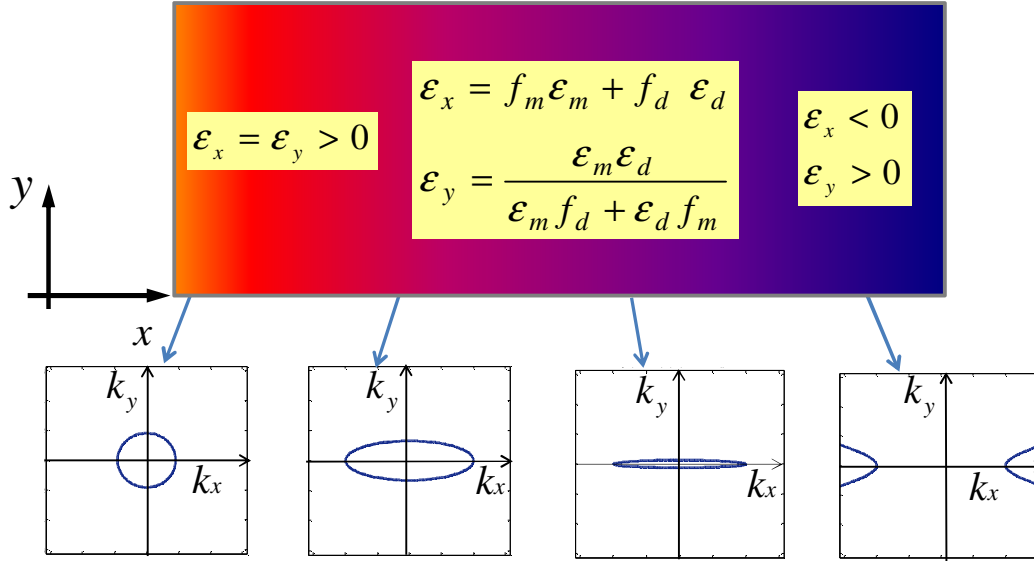


Figure 3. Upper plot: the structure consisting of isotropic material on the left and anisotropic hyperbolic metamaterial on the right. In this structure dielectric permittivity is changed along x direction by changing filling fraction of the metal. Lower plot: evolution of dispersion relation from a circle to an ellipse and then to a hyperbola.

The outstanding question is how to realize such transition metamaterial in practice. Recently, it was shown that strongly anisotropic materials that have negative permittivity component along the direction perpendicular to the interface of the structure and positive component along the interface, so-called hyperbolic metamaterials, the component of the Poynting vector along the interface is opposite to the

direction of the wavevector component along the interface, leading to the effective negative refractive index. Since manipulating magnetic permeability at optical frequencies, although has been demonstrated, is still challenging, this non-magnetic anisotropic metamaterials-based approach to achieving effective negative refractive index is very attractive.

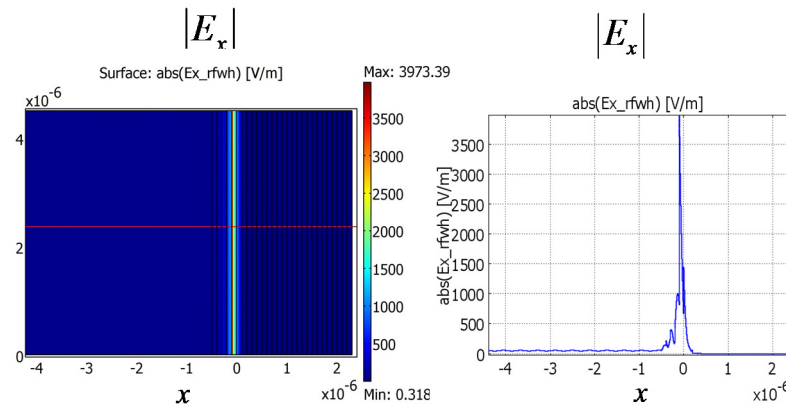


Figure 4. Color map and a line plot demonstrating the effect of anomalous field enhancement near zero-epsilon transition.

In our work, we designed anisotropic transition materials and study the electromagnetic

wave propagation through such structures. Schematic of the structure is as shown in Fig. 3. It has isotropic material on the left and anisotropic hyperbolic material on the right. Varying filling fraction of metal results in variation of ϵ_y . The dispersion relation is circular when $\epsilon_x = \epsilon_y > 0$, (this is the case of an isotropic media). Now slowly decreasing ϵ_x would result in a case where $\epsilon_x, \epsilon_y > 0$ but $\epsilon_x \neq \epsilon_y$ and now the dispersion relation is elliptical in shape. Further decreasing ϵ_x such that $\epsilon_x < 0$ while $\epsilon_y > 0$ would result in a hyperbolic dispersion relation.

A metal-dielectric layered media constituting of metal permittivity ϵ_m and dielectric permittivity ϵ_d of permittivity results in anisotropic structure such that x and y components of the permittivities of the structure are different, that is, $\epsilon_x \neq \epsilon_y$. According to Maxwell Garnett effective medium approximations; ϵ_x and ϵ_y for such a structure are given by

$$\epsilon_x = f_m \epsilon_m + f_d \epsilon_d$$

$$\epsilon_y = \frac{\epsilon_m \epsilon_d}{\epsilon_m f_d + \epsilon_d f_m}$$

where f_m and f_d are filling fractions of metal and dielectric, such that $f_m + f_d = 1$, in our case we consider $\epsilon_m = -3.0 + 0.1i$ and $\epsilon_d = 12$.

Dispersion relation for anisotropic media is given by

$$\frac{k_x^2}{\epsilon_y} + \frac{k_y^2}{\epsilon_x} = \frac{\omega^2}{c^2}$$

where k_x, k_y are projections of k vector on x and y axes respectively; ϵ_x, ϵ_y are permittivity components along x and y axes respectively. Resonant field enhancement in such a medium is shown in Fig. 4.

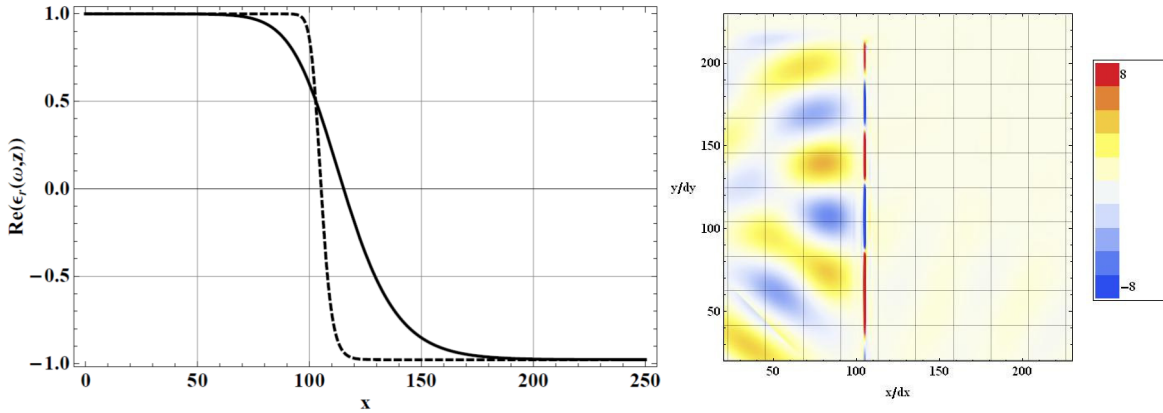


Figure 5. Left: Transition layer profiles. Right: Field distribution of H_x component at the time step $n = t / \Delta t = 3000$ for dashed profile of refractive index.

Finally, while our initial studies were focused on a monochromatic plane wave propagation in transition metamaterials, recently, we developed a finite difference time domain method (FDTD) based code that allows us to study pulse propagation in these structures. In this case, we assume that both dielectric permittivity and magnetic permeability are dispersive. Using our FDTD code, we investigated electromagnetic wave propagation through transition dielectric - metamaterial layer, with two different shapes shown in Fig. 5. Here the discretization steps for space ($\Delta h = \Delta x = \Delta y$) and time (Δt) is chosen

to fulfill the stability requirement of the FDTD method, a so-called Courant condition. An example result of calculations is shown in Fig. 5.

1.3 Instabilities in nonlinear optical MMs

One of the major requirements for the realization of efficiently performing optoelectronics circuits is the ability to buffer optical signals so that the data traffic jams are prevented. An optical buffer is a device that slows down (or even stops) light to store it for a certain period of time. Although several approaches to the realization of such structures have been demonstrated a majority of slow light schemes based on various waveguide geometries are not easily scalable to a chip-size footprint.

It is well known that optical bistability, a phenomenon in which two different values of output power are possible for the same input power, finds numerous applications in optical memory and storage devices. Therefore, realized in compact configuration, it can provide a viable solution for all-optical on-chip storage applications.

Metamaterials (MMs) were shown to enable subwavelength waveguides and cavities – a property that fundamentally differentiates them from conventional materials based light wave components. Therefore, in this work, we investigate the most general solution for wave interactions in positive-negative index MM based nonlinear optical couplers (shown in Fig. 6). It should be mentioned that nonlinear optical couplers made of conventional positive index materials (PIMs) are not bistable (unless some additional components providing optical feedback are introduced). However, in MM based couplers, bistability results from the effective feedback mechanism enabled by the opposing directionality of phase and energy velocities in negative index materials (NIMs). Moreover, such a coupler supports gap solitons—a feature commonly associated with periodic structures. These unusual properties of MM directional couplers form a basis for the development all-optical processing applications, including wavelength converters, flip-flops, and mirrorless lasers.

Our previous studies focused on particular cases of phase-matched symmetric couplers with identical nonlinear properties, and on asymmetric couplers with only one nonlinear channel. In this work, we found a generalized analytical solution in the presence of phase mismatch and for arbitrary values of nonlinear coefficients of both channels. The availability of such a solution enables novel optimized designs of such couplers. In particular, we investigate the effects of bistability and modulational instabilities in positive-negative index based nonlinear optical couplers. Optical bistability is a phenomenon in which two different values of output power are possible for the same input power. This phenomenon finds numerous applications in optical memory and storage devices. It should be mentioned that nonlinear optical

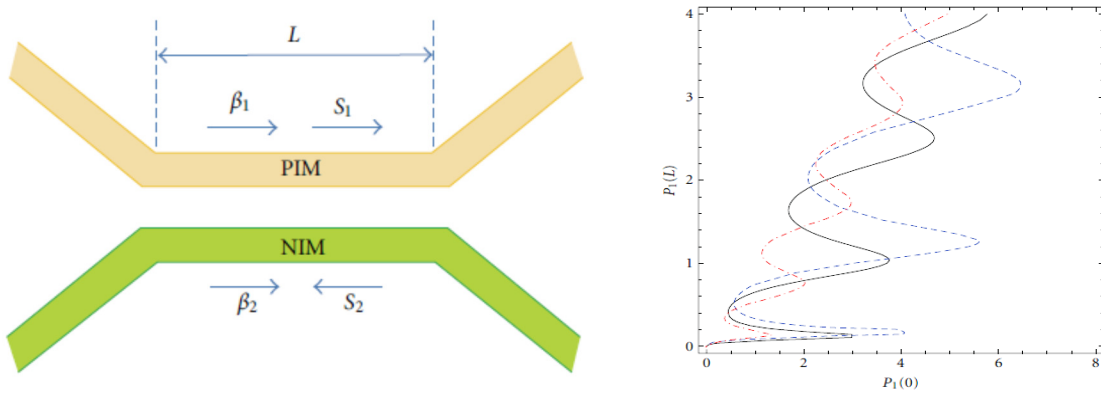


Figure 6. Left: Positive-negative index nonlinear optical coupler. Right: Output power $P_1(L)$ as a function of input power $P_1(0)$ when $\kappa = 8$, $\gamma_1 = \gamma_2 = -5$ (self-defocusing nonlinearity). Solid black curve: $\delta = 0$; dashed blue curve: $\delta = 10$; dot-dashed red curve $\delta = -10$.

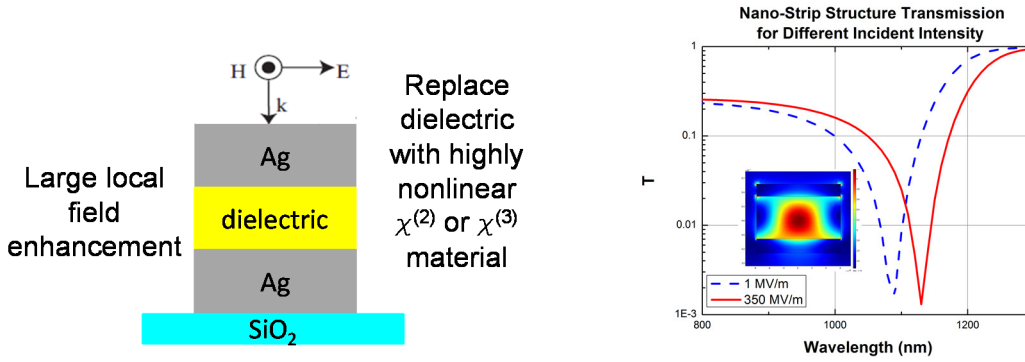


Figure 7. Left: design of nonlinear metamaterials. Right: Transmission as a function of input intensity (inset shows the distribution of refractive index in a nonlinear magnetic metamaterial).

couplers made of conventional positive index materials are not bistable (unless some additional components such as Bragg gratings or mirrors are introduced). However, in MMs-based couplers, bistability results from the effective feedback mechanism enabled by opposing directionality of the wave vector and the Poynting vector in NIMs. Moreover, such a coupler supports gap solitons—a feature commonly associated with periodic structures. These effects have no analogies in conventional couplers. These unusual properties of MM directional couplers form a basis for the development all-optical processing applications, including wavelength converters, flip-flops, and mirrorless lasers. Moreover, MMs allow for ultra-compact (subwavelength) design of such couplers.

We found a generalized analytical solution for nonlinear wave interactions in PIM-NIM couplers in the presence of phase mismatch and for arbitrary values of nonlinear coefficients in both channels. These results offer a practical tool for designing novel MM based couplers based on either double-negative or strongly anisotropic MMs that are likely to enable ultra-compact optical storage and memory components for photonics on a chip applications.

Although the analysis of the PIM-NIM coupler in this work was based on coupled-mode equations assuming effective medium parameters for the dielectric permittivity and magnetic permeability of MMs, in practice such aNIM channel (which is the most challenging part of the proposed device) can be realized in at least two configurations: i) using double-negative resonant MMs, and ii) using strongly anisotropic MM waveguides. Such waveguides were shown to support negative-index propagating modes. For these modes, the wave propagation is in a direction opposite to the phase velocity. As a result, the waveguide behaves as a 2-dimensional counterpart of 3-dimensional negative index material. Such a waveguide can be designed using alternating metal and dielectric subwavelength layers with positive and negative permittivities, respectively. As a nonlinear optical material, we envision incorporate chalcogenide glasses or nonlinear polymers that possess relatively high nonlinear refractive indices.

1.4 Nonlinearly Tunable MMs

Since the first experimental demonstration of optical MMs it became clear that the possibility of controlling of their parameters at the post-fabrication stage — tunability — is one of the key steps toward their practical applications. Indeed, to take full advantage of the designer properties of MMs, a method to tune the electromagnetic response of the MM, preferably over a broad frequency range in as short of time as possible, is required. To date, nearly all demonstrations of actively tuned MMs have been achieved by varying the capacitance of a split ring resonator. This inherently limits the tuning to the magnetic response and to frequencies in the terahertz or lower. The very first designs of tunable MMs in the optical frequency range were based on using nematic liquid crystals. In both works, tunability was achieved by changing the dielectric function of the nematic liquid crystals using linear effects, either using DC voltage or thermal sources.

We investigated a radically different approach that uses strong Kerr nonlinearity of the nematic liquid crystals to control the location of the magnetic and electric resonances in the coupled-nanostrip and fishnet-based MM structures. We studied the interaction of intense fields with such structures which are covered by a nematic liquid crystal, a design that is most realistic and efficient in terms of fabrication and tunability. For example, our numerical simulations performed based on COMSOL Multiphysics platform showed that a 0.3 W/cm^2 CW plane-wave can induce a shift of $\sim 15 \text{ nm}$ in the location of the transmission minimum at the magnetic resonance. This change is comparable to the relative change reported in MMs in the microwave range. Moreover, since the new approach employs the significant near-field enhancement of the electric field, and thus, the localized nonlinear response, the resulting optically-tunable shift is larger than the shift induced with a uniform linear bias-field of a similar magnitude.

Finally, we performed detailed design and optimization of metal-dielectric and all semiconductor-based nonlinear NIMs. To date, nonlinear NIM have not been demonstrated at optical frequencies where they are expected to lead to a number of new phenomena and applications, including backward-phase matching, new regimes of second harmonic generation, and parametric amplification. We progress toward nonlinear NIM fabrication (using focused ion beam lithography) and optical characterization (using z-scan, transmission and reflection measurements, and spectroscopic ellipsometry). The end goal of this project is to demonstrate the first nonlinear NIMs at optical frequencies and use them for sensing, wavelength conversion, and other applications.

Recently, we developed an efficient method for introducing third-order nonlinearities in optical nanostructured materials, including photonic metamaterials. Fig. 7 presents an example of a nonlinear magnetic metamaterial structure (one unit cell of such structure is shown) that was designed and studied using this approach. Our preliminary theoretical results predict a number of novel effects in such nanostructures, such as the higher order resonances shown in Fig. 8. The method uses scalar magnetic field frequency domain formulation; it is shown to produce fast and accurate results without superfluous vector electric field formalism. A standard TM representation using a cubic non-linear susceptibility is problematic due to an intractable implicit equation; our technique alleviates this problem. This new two-dimensional magnetic field formulation was found to exhibit substantially faster performance and converged over a broader range of nonlinearities compared with the three dimensional formulation.

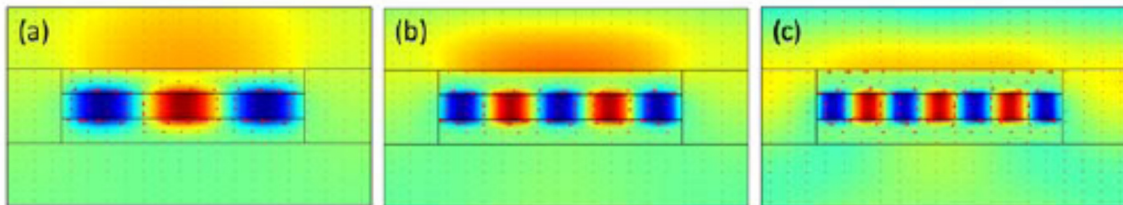


Figure 8. Higher order resonances in metamagnetic plasmonic structure.

1.5 Variable focus nonlinear lenses

By leveraging the capabilities of photonics (speed) and of electronics (compactness), it should be possible to realize high performance integrated opto-plasmonic systems with applications from high bandwidth communications to sensing, and beyond. Such integration requires the availability of ultra-compact, ultra-fast, reconfigurable and tunable photonic components.

We proposed and designed one of such reconfigurable electromagnetic (EM) components with variable focus such that its output field profile can be tuned from an unfocussed beam to a highly localized beam with extended focal region that can be moved from infinity towards the lens surface by changing the

intensity of the incident beam (Fig. 9). Two configurations of such device will be discussed: i) self-focusing beam and ii) all-optically externally controlled focusing.

In cylindrical coordinate system, the space of a concentrator is divided into two regions; the core and the shell. The shell of the concentrator channels the incident field into the core where it is concentrated. The transformation needed to ‘squeeze’ light into the core in reduced parameters can be described by:

$$r' = \begin{cases} R_1 r / R_2 & 0 \leq r \leq R_1 \\ (R_3 - R_1)r / (R_3 - R_2) - (R_2 - R_1)R_3 / (R_3 - R_2) & R_1 \leq r \leq R_3 \end{cases}$$

This transformation is mapped to material tensors as follows:

$$\epsilon_r^{i,j} = \mu_r^{i,j} = \begin{cases} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (R_2 / R_1)^2 \end{bmatrix} & 0 \leq r \leq R_1 \\ \begin{bmatrix} (r+M)/r & 0 & 0 \\ 0 & r/(r+M) & 0 \\ 0 & 0 & N(r+M)/r \end{bmatrix} & R_1 < r \leq R_3 \end{cases}$$

$$\begin{aligned} \text{where } M &= (R_2 - R_1)R_3 / (R_3 - R_2) \\ N &= [(R_3 - R_2) / (R_3 - R_1)]^2 \end{aligned}$$

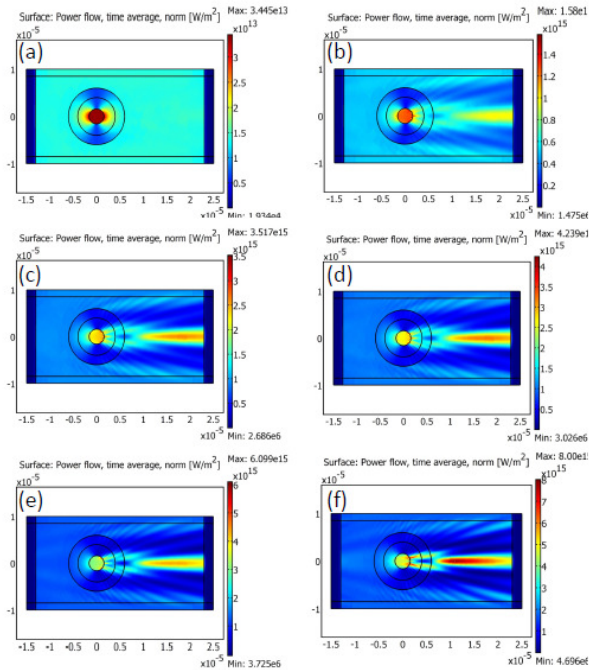


Figure 9. Finite-element method based simulations of variable focus nonlinear lens for different values of input light intensity. Intensity increases from (a) to (f).

Tunable lens proposed here utilizes a nonlinear core element. We use Kerr-type nonlinear medium having a cubic nonlinearity defined by $\chi^{(3)}$. Nonlinear phenomena change the optical properties of the core as the refractive index of the core dependent on the intensity of the incident electric field. In materials that exhibit third-order nonlinearity, index of refraction n can be expressed as $n = n_0 + n_2 I$.

Computation domain is terminated by perfectly matched layers (PMLs) to absorb the scattered field. Structure is illuminated by a TE polarized plane wave incident from left having a wavelength of $1.5 \mu\text{m}$. The core has a nonlinear refractive index as described in the previous section. Our variable focus nonlinear lens based device acts as a concentrator when it is operating in linear regime. As can be seen, all the power is concentrated inside the core region of the device. As the intensity of the incident field is increased, the refractive index of the core increases because of the presence of nonlinearity. Electromagnetic radiation is slowed down inside the core and the part of the wave that travels the farthest is delayed the

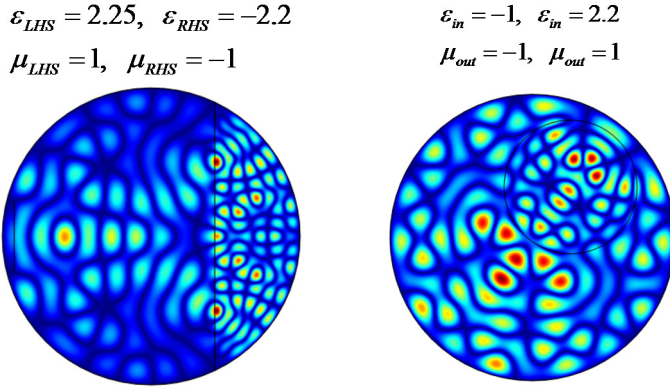


Figure 10. Examples of PIM-NIM cavities that produce scar modes.

most, which results in a net curvature and hence focusing of the field. The device behaves like a lens and focusses field such that the intensity is confined to a narrow, localized spot. The location of that focal spot moves from infinity (linear case) towards the lens as shown in Fig. 9 when intensity of incident field is increased.

We demonstrated a tunable structure which can be switched from the functionality of a concentrator to that of lens which has a variable focus that is tunable

based on the intensity of the incident electric field. By using variable focus nonlinear lens, we can achieve sub-wavelength tunability of the focus point. This device may find applications for the development of ultra-compact optical components for all optical circuits.

1.6 Chaos in metamaterial cavities

As an extension of this project, we considered the wave and ray properties of electromagnetic wave interaction in two-dimensional microcavities that contain a combination of PIM and NIM with negative dielectric permittivity and magnetic permeability. By using a combination of analytic and numerical methods we are able to classify the different types of possible solutions in this type of mixed PIM-NIM regions.

In particular, we investigated the special properties of whispering gallery modes as well as the

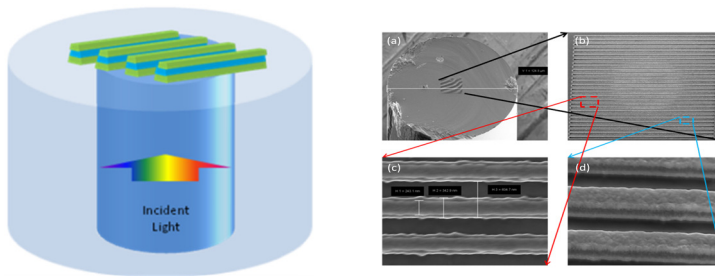


Figure 11. Fiber metamaterials. Left: schematic view. Right: experimental samples.

constructive and destructive interference due to wave refraction/reflection across different refractive media boundaries. We considered closed and open cavities and predict chaotic behavior, and in particular, so-called scar modes (Fig. 10). We explored practical applications of this type of metamaterial micro-cavities, including selective amplification of scar modes and optical nonlinearities.

1.7 Fiber metamaterials

To date, magnetic and negative-index metamaterials at optical frequencies were realized on bulk substrates in the form of thin films with thicknesses on the order of, or less than, optical wavelengths. We designed and experimentally demonstrated, for the first time, fiber-coupled magnetic metamaterials integrated on the transverse cross-section of an optical fiber (Fig. 11). Such fiber-metamaterials integration may provide fundamentally new solutions for photonic-on-a-chip systems for sensing, subwavelength imaging, image processing, and biomedical applications.

III. Publications and presentations

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